Source Mechanisms of Mine-Related Seismicity,
Savuka Mine, South Africa

by Jordi Julià, Andrew A. Nyblade, Ray Durrheim,* Lindsay Linzer,
Rengin Gök, Paul Dirks, and William Walter

Abstract We report full moment tensor solutions for 76 mine tremors with moment
magnitudes ($M_w$) between 0.5 and 2.6 recorded by a network of 20 high-frequency
gephones in a deep gold mine in South Africa. Source mechanisms convey important
information on how in-mine stresses are relaxed, and understanding the nature of such
mechanisms is essential for improving our assessment of rock mass response to
mining. Our approach has consisted of minimizing the L2 norm of the difference
between observed and predicted $P$, $SV$, and $SH$ spectral amplitudes, with visually
assigned polarities, to constrain all six independent components of the seismic
moment tensor. Our results reveal the largest principal stresses in the mine are com-
pressive, oriented near vertically, and relaxed through a mix of volumetric closure and
normal faulting, consistent with a gravity-driven closure of the mined-out areas. Pre-
vious moment tensor studies in deep mines had suggested that the distribution of seis-
mic sources in terms of the volumetric-shear mix was bimodal. A bimodal distribution
is compatible with our moment tensor solutions only for moment magnitudes above
2.2. Events in the $0.5 < M_w < 2.2$ moment magnitude range display a continuous
distribution of their volumetric-shear mix.

Online Material: Focal parameters for mine tremors at Savuka.

Introduction

An early debate in mining seismology concerned the
need for invoking nondeviatoric failure mechanisms to ex-
plain the radiation patterns generated by mine-induced seis-
mic events (see, e.g., Wong and McGarr, 1990, and ref-
ences therein). This debate was settled in the early 1990s by
McGarr (1992a,b), who convincingly demonstrated the
necessity of an implosive component to fully explain the
radiation patterns of 7 events observed in deep, tabular gold
mines in two mining districts in South Africa, and by Feigner
and Young (1992), who showed a significant volumetric
component was required to explain the radiation patterns
of 19 microseismic events generated during a mine-by ex-
periment at the Underground Research Laboratory (URL)
in Manitoba, Canada. In his study of South African source
mechanisms McGarr (1992b) noted that the distribution of
moment tensor solutions in the deep mines was bimodal:
the mine tremors were either pure shear failures, or had a
ratio of coseismic closure to average shear slip around 0.71.
McGarr (1992b) also noted that the moment tensor solutions
reported by Feigner and Young (1992) had a more con-
tinuous distribution of source mechanisms in terms of their
volumetric-shear mix.

A number of differences between those two studies
could explain the apparent paradox posed by the source-
mix distribution of the focal mechanisms. First, the size
of the database utilized by Feigner and Young (1992) was
significantly larger (33 events) than that utilized by McGarr
(1992b) (10 events), and this difference makes the first study
more relevant statistically. Second, the mining conditions
under which the seismic events were induced were radically
different; the deep gold mines described by McGarr (1992b)
have a maximum principal stress (in absolute sense) that is
negative (i.e., compressive) and near-vertical, which could
explain the predominance of implosive sources; the URL
in Canada is characterized by a maximum principal stress
that is positive and subhorizontal (Feigner and Young, 1992),
which could explain the existence of tensile sources. Third,
the moment magnitude range for the deep-mine events is
$1.9 < M_w < 3.3$, while the magnitude range for the URL
events is $-3.3 < M_w < -2.3$. The differences could thus
be the result of a magnitude-dependent distribution for the
source mix. And fourth, the observations utilized in both
studies to infer the moment tensor solutions were different

*Also at School of Geosciences, University of the Witswatersrand, Wits
2050, South Africa.
and could have biased the solutions in different ways; McGarr (1992b) utilized select waveform amplitudes in the time domain, while Feigner and Young (1992) utilized spectral amplitudes with polarities attached.

More recently, a moment tensor study of six shallow events, with moment magnitudes between 1.6 and 1.8, in a coal mine in Utah revealed a continuous variation of the source mix (Fletcher and McGarr, 2005). The moment tensor solutions were obtained from select time-domain amplitudes, as in McGarr (1992b), thus demonstrating that the bimodal distribution reported in McGarr (1992b) did not reflect any bias introduced by the inversion method. The lack of overlap in the magnitude range, the small size of the data set, and the different mining conditions (shallow versus deep), however, can still explain the observed differences in the source-mix distributions.

In this study, we report moment tensor solutions for 76 well-recorded seismic events, with moment magnitudes in the 0.5–2.6 range and recorded in a deep gold mine in South Africa (Savuka mine, Carletonville mining district). The moment tensor solutions have been obtained by inverting spectral amplitudes with polarity attached and verified in the time domain with synthetic seismograms. Our results demonstrate that the source-mix distribution is continuous under deep mining conditions in the $0.5 < M_w < 2.2$ moment magnitude range. At larger magnitudes our results are less conclusive but still compatible with a bimodal distribution of the volumetric-shear mix, as reported by McGarr (1992b).

Data

The data set utilized in this study has been obtained from a network of 20 seismic stations deployed underground by Integrated Seismic Systems International (ISSI) to monitor mine-related seismic activity at the AngloGoldAshanti Savuka gold mine, South Africa (Fig. 1). The network consists of a combination of three-component sensor types: the G4.5 geophone, with a flat response in velocity between 4

![Figure 1](source.png)

Figure 1. Plan of Savuka mine, South Africa, with the location of the in-mine geophones (circles) and the 100 seismic events (gray squares) considered in this study. The black circles denote the stations utilized for moment tensor inversion and the gray circles indicate the stations that could not be utilized. The thin solid lines delineate the outline of the mine, while the shaded areas represent the areas that were mined during 2007. The hypocentral locations were obtained from ISSI.
and 2000 Hz and a sensitivity of 2.8 V/m/sec; the G14 geophone, with flat response in velocity between 12 and 2000 Hz and a sensitivity of 80 V/m/sec; and the G28 geophone, with a flat response in velocity between 28 and 2000 Hz and a sensitivity of 15 V/m/sec. The seismic stations are operated at sampling rates ranging from 2000 to 10,000 samples per second, with timing synchronized through common Global Positioning System time keeping. Except for one surface sensor, the sensors are deployed at depth along the two gold-bearing horizons being mined: the Ventersdorp Contact Reef (VCR) and the Carbon Leader Reef (CLR). The VCR is mined at depths of around 3000 m, with Ventersdorp Lavas in the hanging wall and a thin layer of shale in the footwall. The CLR is mined at depths of around 3500 m and is separated from the shallower VCR by an ~500 m thick layer consisting predominantly of quartzite.

Data Selection

A total of 11,224 mine tremors, with local magnitudes between -3.4 and 4.4, were cataloged by ISSI during 2007 through the Savuka mine in-mine network. This large catalog includes events induced by mine activity at the Savuka mine itself, as well as events induced in nearby mines. Good azimuthal coverage is critical for accurately mapping the seismic radiation patterns of the recorded mine tremors, so a selection was made for those events with locations falling near or within the volume covered by the in-mine network. Furthermore, the moment tensor inversion procedure employed in this study assumes the propagating medium is a whole space of constant wave speed. This is a reasonable assumption, as the seismic waves from the selected events are expected to propagate through the quartzite layer separating the two gold-bearing horizons. Nonetheless, to ensure that the selected seismic phases propagate at constant wave speed, Wadati diagrams were constructed for each event recorded at as many as 16 stations, randomly selected by ISSI to label the event. Note that two stations, SAV34 and SAV64, have been left out of the Wadati line. The $V_p/V_s$ ratio, origin time correction, and regression coefficient from the least-squares fit to the data points (not including the rejected stations) is shown in the plot. Confidence bounds are given at the 95% confidence level.

Sensor Orientation

Sensor orientations were verified by comparing theoretically rotated waveforms with empirically rotated waveforms (Fig. 4). Theoretical rotations were performed through the rotation angles calculated as

\[
\tan \varphi = \frac{(y_S - y_0)/(x_S - x_0)}{x};
\]

\[
\cos \phi = \frac{(z_S - z_0)/((x_S - x_0)^2 + (y_S - y_0)^2)^{1/2}}{x} + (z_S - z_0)^2 {1/2},
\]

where $(x_S, y_S, z_S)$ and $(x_0, y_0, z_0)$ are the station and source coordinates, respectively, with $x$ being north, $y$ being east, and $z$ being up, and $\phi$ and $\varphi$ are the takeoff angle and azimuth, respectively. The empirical rotation angles, on the other hand, were obtained from the eigenvectors associated with the eigenvalues of the covariance matrix

\[
\begin{bmatrix}
\text{Var}[x] & \text{Cov}[x, y] & \text{Cov}[x, z] \\
\text{Cov}[x, y] & \text{Var}[y] & \text{Cov}[y, z] \\
\text{Cov}[x, z] & \text{Cov}[y, z] & \text{Var}[z]
\end{bmatrix}
\]

recordings; for SAV34, the discrepancy could be caused by either a misidentification of the $S$-wave arrival in the north component or a deviation from propagation at uniform wave speed.

Figure 2. Wadati diagram for event 2007.01.03.18.31.21.223. The reference time on the horizontal axis is the trigger time from one of the recording stations, randomly selected by ISSI to label the event. Note that two stations, SAV34 and SAV64, have been left out of the Wadati line. The $V_p/V_s$ ratio, origin time correction, and regression coefficient from the least-squares fit to the data points (not including the rejected stations) is shown in the plot. Confidence bounds are given at the 95% confidence level.
defined by Montalbetti and Kanasewich (1970), where
\[
\text{Var}[x] = \frac{\sum (x_i - \bar{x})^2}{(n-1)}, \quad \text{Cov}[x, y] = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1)},
\]
with * denoting the average within the P-wave window of the in-mine recordings and \( n \) denoting the number of data points. Realize that this polarization filter only yields the directions of the P, SV, and SH axes and cannot resolve the sense of motion due to the symmetry of the covariance matrix with respect to reflections about the origin of coordinates.

The sensor orientation could only be verified for 13 stations out of the 20 in the network, as 7 of them had at least one component failing for the entire recording period, which prevented the rotation into the local ray-coordinate system. Additionally, we observed that the theoretically and empirically rotated waveforms did not compare well for 7 of those 13 stations. Fortunately, corrections could be obtained for 4 of them through rotations around the vertical axis, yielding a total of 10 stations available for moment tensor inversion (Table 1). However, an ambiguity of 180° around the vertical axis is still possible. Indeed, we observed that waveforms recorded at stations SAV79 and SAV81 were, for some events, more consistent with the moment tensor solutions when rotated by 180°. Therefore, these stations were also not used to constrain focal mechanisms.

A final event selection was done after sorting the events by the number of recording stations with verified sensor orientations, in descending order, and selecting the top 100. The hypocentral locations and origin times for the selected events are listed in Table E1 (available in the electronic edition of BSSA).

**Spectral Amplitudes**

Spectral amplitudes for the P, SV, and SH pulses were obtained in the time domain from the theoretically rotated waveforms through the integrals of squared velocities and squared displacements according to (Trifu et al., 2000).
\[ u = \frac{2S_{D2}^{3/4}}{S_{V2}}, \]  

where \( u \) is the spectral amplitude, \( S_{D2} = \int D^2(t) \, dt \), \( S_{V2} = \int V^2(t) \, dt \), \( D \) is displacement, \( V \) is velocity, and the integrals are evaluated within finite time windows. Following Trifu et al. (2000), \( P \) waveforms were windowed between the \( P \)-wave and the \( S \)-wave arrivals and \( S \) waveforms were windowed between the \( S \)-wave arrivals and a time window two times the \( S - P \) travel-time difference in length. Polarieties for the spectral amplitudes were visually assigned by comparing the low-pass filtered instrument response in the time domain to the observed low-pass filtered \( P \)- and \( S \)-wave pulses (Fig. 5).

Although the spectral amplitudes are evaluated in the time domain, they are equivalent to measuring the height of the spectral plateau in the frequency domain (Urbancic et al., 1996). Consequently, the low-pass filter must go below the corner frequency to make the observed pulse shape similar to the shape of the instrument response in the time domain. Typically, we were successful in assigning polarities after low-pass filtering below 10 Hz using two passes of a third order Butterworth filter. However, difficulties were sometimes experienced with the 14 and 28 Hz geophones and with stations very close to the source. In such cases, we generally succeeded by low-pass filtering below 20 or 40 Hz, which suggests that the frequency content of the filtered waveform was still within the flat part of the source spectrum. Time shifts between the \( SV \) and \( SH \) pulse, probably due to anisotropic effects, were observed during this process, but because our moment tensor solutions involve only spectral amplitudes, the time shifts have no effect on our moment tensor solutions.

### Table 1

<table>
<thead>
<tr>
<th>ID</th>
<th>South (m)</th>
<th>West (m)</th>
<th>Down (m)</th>
<th>Rotation (?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAV08</td>
<td>27950</td>
<td>-39827</td>
<td>1869</td>
<td>?</td>
</tr>
<tr>
<td>SAV21</td>
<td>27816</td>
<td>-40454</td>
<td>226</td>
<td>?</td>
</tr>
<tr>
<td>SAV27</td>
<td>28676</td>
<td>-39582</td>
<td>2003</td>
<td>?</td>
</tr>
<tr>
<td>SAV29</td>
<td>29617</td>
<td>-40545</td>
<td>3649</td>
<td>-90</td>
</tr>
<tr>
<td>SAV34</td>
<td>28330</td>
<td>-40660</td>
<td>2142</td>
<td>180</td>
</tr>
<tr>
<td>SAV35</td>
<td>29024</td>
<td>-40247</td>
<td>2285</td>
<td>55</td>
</tr>
<tr>
<td>SAV36</td>
<td>27958</td>
<td>-40226</td>
<td>2960</td>
<td>ok</td>
</tr>
<tr>
<td>SAV40</td>
<td>27738</td>
<td>-39822</td>
<td>2964</td>
<td>ok</td>
</tr>
<tr>
<td>SAV61</td>
<td>27854</td>
<td>-40568</td>
<td>2010</td>
<td>ok</td>
</tr>
<tr>
<td>SAV77</td>
<td>28220</td>
<td>-39753</td>
<td>3089</td>
<td>90</td>
</tr>
<tr>
<td>SAV79</td>
<td>28109</td>
<td>-40220</td>
<td>3733</td>
<td>ok’</td>
</tr>
<tr>
<td>SAV80</td>
<td>29114</td>
<td>-41600</td>
<td>3657</td>
<td>ok</td>
</tr>
<tr>
<td>SAV81</td>
<td>29000</td>
<td>-41638</td>
<td>3493</td>
<td>ok’</td>
</tr>
</tbody>
</table>

*Not utilized due to orientation ambiguity.*

### Inversion Method

Estimates for the six independent components of the seismic moment tensor were obtained by inverting \( P \), \( SV \), and \( SH \) spectral amplitudes with polarities attached. As explained before, the ray paths for the selected seismic events are mostly contained within the \(~500\) m thick layer of quartzite separating the two gold-bearing reefs, so the
forward problem can be safely formulated for a homogenous whole space. Following Trifu et al. (2000), the forward problem can be expressed as

\[ \mathbf{u} = c \mathbf{F} \mathbf{M}, \]  

(4)

where \( \mathbf{u} \) is the vector of spectral displacements in the local ray-coordinate system, \( c = 1/(4\pi \rho v^2)R \) with \( v \) being either the \( P \)- or \( S \)-wave velocity, \( R \) the hypocentral distance, \( \rho \) the density, and \( \mathbf{M} \) is the matrix defining the seismic moment tensor in the geographic system. \( \mathbf{F} \) is the excitation matrix given by

\[
\mathbf{F}^P = \begin{bmatrix}
\sin^2 \phi \cos^2 \varphi & 1/2 \sin^2 \phi \sin 2\varphi & 1/2 \sin 2\phi \cos \varphi \\
1/2 \sin^2 \phi \sin 2\varphi & \sin^2 \phi \sin^2 \varphi & 1/2 \sin 2\phi \sin \varphi \\
1/2 \sin 2\phi \cos \varphi & 1/2 \sin \phi \sin \varphi & \cos^2 \phi 
\end{bmatrix},
\]

\[
\mathbf{F}^{SV} = \begin{bmatrix}
1/2 \sin 2\phi \cos^2 \varphi & 1/4 \sin 2\phi \sin 2\varphi & \cos^2 \phi \cos \varphi \\
1/4 \sin 2\phi \sin 2\varphi & 1/2 \sin 2\phi \sin^2 \varphi & \cos^2 \phi \sin \varphi \\
- \sin^2 \phi \cos \varphi & \sin^2 \phi \sin \varphi & - 1/2 \sin 2\phi 
\end{bmatrix},
\]

\[
\mathbf{F}^{SH} = \begin{bmatrix}
-1/2 \sin \phi \sin 2\varphi & - \sin \phi \sin^2 \varphi & - \cos \phi \sin \varphi \\
\sin \phi \cos^2 \varphi & 1/2 \sin 2\phi \sin \varphi & \cos \phi \cos \varphi \\
0 & 0 & 0
\end{bmatrix},
\]

where \( \phi \) and \( \varphi \) are, again, the takeoff angle and azimuth, respectively.

Because of the symmetry of the moment tensor \( \mathbf{M} \), equation (4) can be rewritten as

\[ u_j = \sum_k c_j^i f_k^j m_k, \quad i = P, SV, SH, \quad j = 1, \ldots, n, \]

(6)

where \( n \) is the number of data points, \( m_1 = M_{11}, \)

\[ m_2 = M_{12} = M_{21}, \quad m_3 = M_{22}, \quad m_4 = M_{13} = M_{31}, \quad m_5 = M_{23} = M_{32}, \quad m_6 = M_{33} \]

and where \( f_1 = F_{11}, \quad f_2 = F_{12} + F_{21}, \quad f_3 = F_{22}, \quad f_4 = F_{13} + F_{31}, \quad f_5 = F_{23} + F_{32}, \quad \) and \( f_6 = F_{33} \). Equation (6) defines a linear problem, which is solved for the moment tensor elements \( m_k \) with no constraints (i.e., full moment tensor solution) after a singular value decomposition (SVD) of the matrix \( c_j^i f_k^j \) (see, e.g., Menke, 1984). Because the only moment tensor solutions that do not require a truncation of the singular value spectrum are capable of independently constraining all moment tensor components, in practice, the moment tensor solutions reported in this study minimize the difference between observations and predictions in a least-squares sense.

The \( c \) factor in equations (4) and (6) does not include any corrections for anisotropy or attenuation. As noted before, time shifts were observed between the \( SV \) and \( SH \) amplitudes, which we attribute to anisotropic propagation effects due to horizontal layering. The \( SV \) and \( SH \) components coincide with the \( S1 \) and \( S2 \) components in a transversely isotropic medium, which may delay the pulses with respect to each other but will not introduce any pulse doubling (Backus, 1962). Because the inversions are performed in the frequency domain, where phase information is dropped (except for polarity), we do not believe that anisotropy strongly maps into our solutions. Attenuation, on the other hand, could potentially have a stronger effect. As shown later, our reported moment tensor solutions correctly predict the polarities of the observed amplitudes, so our mechanism types should not be strongly affected by attenuation effects. On the other hand, attenuation corrections would make the observed amplitudes bigger, and our moment tensor sizes could be somewhat underestimated.

Formal uncertainties for the components of the seismic moment tensor are not reported in this study. In theory, the linearity of the forward problem defined by equation (6) allows for the matrix of covariance of the spectral amplitudes to be easily propagated into uncertainties of the moment tensor components (e.g., Menke, 1984). Fletcher and McGarr (2005), for instance, assign a standard deviation to the observed amplitudes equal to 20% of the observed amplitude and propagate the uncertainties into the moment tensor solutions. This approach is successful in conveying the relative accuracy among the moment tensor components, but we find it is somewhat arbitrary and statistically misleading. A meaningful matrix of covariance for the spectral amplitudes should include uncertainties in the amplitude measurement as well as uncertainties in the velocities of the propagating medium, sensor orientations, and event locations. Our approach has been to conduct numerical experiments in an attempt to falsify those observations that are critical to our study. As shown later, we will be interested to know how
2.69 g/cm$^3$. Only those moment tensor solutions that correctly predict $P$, $SV$, and $SH$ polarities for all the waveforms were accepted (the polarity requirement was relaxed for small amplitudes, which probably represent null measurements), and the quality of the solutions was assessed through the condition number and the correlation between observed and synthetic amplitudes. The accepted moment tensor solutions, totaling 76, and the corresponding condition numbers and correlation coefficients are listed in Table E2 (available in the electronic edition of *BSSA*).

The condition number is defined as the ratio between the smallest and largest singular values in the SVD and is a measure of how robustly constrained the moment tensor components are. A condition number of 1 indicates that all moment tensor components are equally well constrained, while a condition number of 0 indicates that there is at least one component not resolved and that the solution is nonunique. For our accepted moment tensor solutions the condition numbers range between 0.04 and 0.19, just one to two orders of magnitude smaller than the ideal value. As discussed later, the reported moment tensor solutions define a consistent radiation pattern that agrees well with the stress pattern of the mine, suggesting this range of values reflects that the solutions are well constrained. The reported condition numbers should then be taken as a measure of the relative robustness among the solutions.

The correlation coefficients were obtained from the cross correlation between observed and synthetic waveforms and are a measure of the similarity between observations and predictions. Synthetic seismograms were computed from the spectral amplitudes as

$$u(t) = ud(t - R/v),$$

where $u(t)$ is the displacement vector in the time domain, $u$ is the spectral amplitude from equation (4), $\delta(t)$ is a unit-area Dirac’s delta function, $v$ is either the $P$- or $S$-wave velocity, and $R$ is the hypocentral distance, then differentiated to obtain velocities and finally convolved with the instrument response. The correlation coefficients were obtained as

$$\text{cor} = \frac{1}{3}[C(u_{\text{obs},P}, u_{\text{syn},P}) + C(u_{\text{obs},SV}, u_{\text{syn},SV}) + C(u_{\text{obs},SH}, u_{\text{syn},SH})],$$

where $u_{\text{obs}}$ stands for observation, $u_{\text{syn}}$ for synthetic, and where

$$C(f, g) = \max \left[ \int f(t + \tau)g(t) \, dt \right] \left[ \int f(t)g(t) \, dt \right].$$

where $\tau$ is the time shift, after low-pass filtering both observations and synthetics generally below 10 Hz (20 or 40 Hz for the 14 and 28 Hz instruments). Note that a coefficient of 1.0 implies a perfect agreement between observations and

---

**Figure 5.** Assignment of polarity for the $P$, $SH$, and $SV$ amplitudes recorded at station SAV61 and corresponding to event 2007.02.01.18.08.14.177. The top trace is the instrument response in the time domain (positive polarity) normalized to unit amplitude, and the traces below are the $P$, $SH$, and $SV$ components, respectively. All traces have been low-pass filtered below 10 Hz. The similarity of the impulse response to the time (s) normalized to unit amplitude, and the traces below are the $P$, $SH$, and $SV$ components, respectively.

well constrained the isotropic part of our moment tensor solutions are. The observed spectral amplitudes will then be reinverted for a purely deviatoric source in order to assess the necessity of an isotropic component in the moment tensor solutions. This new suite of inversions will be conducted through the same system defined in equation (6) with the additional constraint that the trace of the moment tensor be zero,

$$m_1 + m_3 + m_6 = 0.$$ (7) 

**Moment Tensor Solutions**

We have applied the inversion of spectral amplitudes with polarity attached described in the previous section to the 100 events selected for this study, after assuming a whole space of $V_P = 6.0$ km/sec, $V_S = 3.70$ km/sec, and $\rho =$

---

**Table E2** (available in the electronic edition of *BSSA*).
predictions, while larger and smaller values indicate the amplitudes have been either overpredicted or underpredicted, respectively. The correlation coefficients corresponding to our accepted moment tensor solutions range between 0.54 and 1.18, and we found that even coefficients as small as 0.54 give a satisfactory visual match between observations and predictions. As with the condition numbers, the values of the correlation coefficient should be used as a measure of the relative goodness of fit of the solutions. In the following, we discuss a few moment tensor solutions in detail.

Event 2007.02.21.18.21.56.591 (Mn 1.6)

Figure 6 displays the moment tensor inversion results for this event, along with the coverage of the focal sphere. Spectral amplitudes for this event were measured for P, SV, and SH arrivals recorded at six in-mine seismic stations and the polarities were visually assigned for each of them, as described before. The coverage of the focal sphere is characteristic of mining environments and reflects the lack of station coverage ahead of the advancing mine stopes. The measured P- and S-wave amplitudes, however, had both positive and negative polarities, which suggests that more than one energy lobe in the radiation pattern has been sampled. The moment tensor solution obtained for this event is 

\[ m_{xx} = -1.25 \times 10^{11} \text{ Nm}, \quad m_{xy} = 0.74 \times 10^{11} \text{ Nm}, \quad m_{yz} = 0.09 \times 10^{11} \text{ Nm}, \quad m_{xz} = 1.20 \times 10^{11} \text{ Nm}, \quad m_{yz} = 0.55 \times 10^{11} \text{ Nm}, \quad m_{zz} = -2.66 \times 10^{11} \text{ Nm} \]

and has a large condition number around 0.16 and a correlation coefficient around ~0.70, which is in the middle-to-low portion of the observed range and offers a satisfactory visual match between predictions and observations.

Further insight can be gained from the eigenvalues (\( \sigma_i \)) and eigenvectors (\( e_i \)) associated to the moment tensor solution. The eigenvalues are indicative of the compressional (negative) or tensitional (positive) character of the principal stresses, and the direction is given by the corresponding eigenvectors (e.g., Lay and Wallace, 1995). For this moment tensor solution, we obtain \( \sigma_1 = -3.35 \times 10^{11} \text{ Nm} \), \( \sigma_2 = -1.22 \times 10^{11} \text{ Nm} \), and \( \sigma_3 = 0.74 \times 10^{11} \text{ Nm} \), and \( e_1 = (0.49, 0.04, -0.87) \), \( e_2 = (-0.72, 0.57, -0.38) \), and \( e_3 = (-0.49, -0.82, -0.31) \). The largest principal stress is
thus compressive and oriented near vertically (deflection from the vertical is \(\sim 29^\circ\)), consistent with the stress conditions expected in a deep mine.

All moment tensors can be decomposed into isotropic and deviatoric components, with the isotropic eigenvalue given by \(\text{tr}(\sigma_i) / 3\), where \(\text{tr}(\sigma_i) = \sigma_1 + \sigma_2 + \sigma_3\) is the trace, and the deviatoric eigenvalues given by \(\sigma^D_1 = \sigma_j - \text{tr}(\sigma_i) / 3\) (e.g., Jost and Herrman, 1989). For this particular moment tensor solution \(\text{tr}(\sigma_i) = -3.83 \times 10^{11}\) N m, representative of an implosive volumetric source. The deviatoric eigenvalues are \(\sigma^D_1 = -2.07 \times 10^{11}\) N m, \(\sigma^D_2 = 0.06 \times 10^{11}\) N m, and \(\sigma^D_3 = 2.01 \times 10^{11}\) N m, which are well approximated by a double-couple force equivalent (e.g., Jost and Herrmann, 1989). The eigenvectors of the deviatoric moment tensor are the same as those for the general moment tensor, so the double-couple solution has a subvertical pressure axis and a subhorizontal tension axis characteristic of normal faulting. The decomposition of the moment tensor solution for this event thus indicates that stresses have been relaxed through normal faulting and coseismic closure (McGarr, 1992a).

Event 2007.02.01.49.31.639 (\(M_w\) 1.1)

More generally, however, we obtain deviatoric moment tensors that do not follow a clear double-couple pattern, and Figure 7 displays the moment tensor inversion results and the coverage of the focal sphere for one such event. All \(P\)-wave amplitudes are negative, although a polarity could not be assigned for SAV35 due to its small amplitude, and polarities of both signs are found among the \(S\)-wave amplitudes. The moment tensor solution obtained for this event is \(m_{xx} = -2.48 \times 10^{10}\) N m, \(m_{xy} = 1.47 \times 10^{10}\) N m, \(m_{yx} = -1.98 \times 10^{10}\) N m, \(m_{yy} = 1.50 \times 10^{10}\) N m, \(m_{zz} = -1.78 \times 10^{10}\) N m, and \(m_{zz} = -4.33 \times 10^{10}\) N m and has a large condition number of \(\sim 0.18\) and a correlation coefficient of 0.88,

Figure 7. Same caption as in Figure 6, but for event 2007.02.01.49.31.639. The \(P\)-wave and \(SH\)-wave spectral amplitudes for station SAV35 and SAV34, respectively, were not included in the inversion due to difficulties in assigning a polarity, but the corresponding synthetic seismograms predicted by our moment tensor solution are shown as a gray, dotted line.
which, once more, offers a satisfactory visual match between observations and predictions.

The eigenvalues for the seismic moment tensor for this event are \( \sigma_1 = -6.36 \times 10^{10} \text{ N m} \), \( \sigma_2 = -1.77 \times 10^{10} \text{ N m} \), and \( \sigma_3 = -0.67 \times 10^{10} \text{ N m} \), and the corresponding eigenvalues are \( \mathbf{e}_1 = (0.47, -0.46, -0.75) \), \( \mathbf{e}_2 = (-0.72, 0.30, -0.63) \), and \( \mathbf{e}_3 = (-0.51, -0.84, 0.20) \), which again reveal a compressive and subvertical maximum principal stress (deflection from vertical is \( \sim 41^\circ \)). The trace of the moment tensor is \( -8.80 \times 10^{10} \text{ N m} \), suggestive of an implosive volumetric contribution and consistent with the negative polarities observed for all \( P \)-wave amplitudes. The deviatoric eigenvalues are \( \sigma_1^D = -3.43 \times 10^{10} \text{ N m} \), \( \sigma_2^D = 1.16 \times 10^{10} \text{ N m} \), and \( \sigma_3^D = 2.27 \times 10^{10} \text{ N m} \), which clearly do not match the double-couple pattern. The deviatoric moment tensor can further be decomposed into major and minor double couples (e.g., Jost and Herrmann, 1989); for this particular event, one possible decomposition would be

\[
\begin{bmatrix}
-3.43 \\
1.16 \\
2.27
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-1.16 \\
1.16 \\
0.00
\end{bmatrix} + \begin{bmatrix}
-2.27 \\
0.00 \\
2.27
\end{bmatrix}
\]

(11)

in \( 10^{10} \text{ N m} \). This decomposition would explain the observed radiation pattern through two normal faults, with a common \( P \) axis and two corresponding \( T \) axes forming an angle of \( 90^\circ \), and a coseismic volumetric closure.

The deviatoric moment tensor can be decomposed in multiple ways and the choice of the decomposition type is arbitrary (see, e.g., Jost and Herrmann, 1989). Another common choice is the decomposition into a compensated linear vector dipole (CLVD) and a double couple, which can be useful in identifying pillar failures (the force equivalent is the compression CLVD combined with an isotropic implosion that annihilates the \( P \)-wave compressions in the directions perpendicular to the principal axis of the CLVD) (e.g., Sileny and Milev, 2006, 2008). The decomposition utilized in equation (11) was the one favored in McGarr (1992a,b), and it has been chosen here for consistency with those studies.

Discussion and Conclusions

The 76 moment tensor solutions obtained in this study reveal that the largest (in an absolute sense) principal stresses are generally negative (i.e., compressional) and oriented subvertically (see Table E3, available in the electronic edition of BSSA). This is in agreement with the expected principal stresses in the deep mine being mainly caused by gravitational forces attempting to close the mined-out areas. Indeed, \textit{in-situ} measurements in the East Rand Proprietary mine (ERPM), also in the Witwatersrand basin, show that the maximum principal stress is compressional and oriented at an angle between the vertical and the normal to the gold-bearing horizon (which dips around \( 30^\circ \)) to the south-southwest in the ERPM area (Pallister \textit{et al.}, 1970). If a decomposition of the deviatoric moment tensor into two double couples as in equation (11) is chosen, then the solutions listed in Table E3 indicate that deep-mine stresses are relaxed through a combination of coseismic volumetric closure and normal faulting.

We have obtained isotropic (\( m^I \)), deviatoric (\( m^D \)), total (\( m^T \)) scalar moments, and moment magnitudes from our moment tensor solutions. Assuming that the eigenvalues of the general moment tensor are given by \( \sigma_1, \sigma_2, \) and \( \sigma_3 \), then the scalar moments are defined as (Bowers and Hudson, 1999)

\[
m^I = |1/3 \text{tr}(\sigma_i)|,
\]

\[
m^D = \max(|\sigma_j^D|), \quad \text{and} \quad m^T = m^I + m^D,
\]

(12)

respectively, where \( \text{tr}(\sigma_i) = \sigma_1 + \sigma_2 + \sigma_3 \) and \( \sigma_j^* = \sigma_j - \text{tr}(\sigma_i)/3 \). The moment magnitude is obtained from the total scalar moment as (Hanks and Kanamori, 1979)

\[
M_w = 2/3(\log_{10}(m^T) - 9.1),
\]

(13)

where \( m^T \) is expressed in newton meter. The values for the scalar moments and moment magnitudes can be found in Table E4 (available in the electronic edition of BSSA).

A careful inspection of our moment tensor solutions (Table E4) reveals that a total of 66 solutions have a significant isotropic moment representing more than 10% of the total moment (computed as \( m^I/m^T \times 100 \)). Before drawing any conclusions about the volumetric-shear mix, however, it is important to assess how well constrained the volumetric components are, and, as explained before, this is investigated by rerunning the moment tensor inversions after imposing a purely deviatoric moment tensor solution (equation 7). This is illustrated in Figure 8 through event 2007.03.04.06.16 .04.098, where low-frequency synthetic seismograms predicted by the full and constrained moment tensor solutions are compared to the corresponding observed traces. Both moment tensor solutions correctly predict the polarities for the \( P, SV, \) and \( SH \) amplitudes, and the correlation coefficients between observed and predicted amplitudes are similar, 0.86 for the full solution and 0.77 for the constrained solution. The condition number of the full moment tensor solution is around 0.15.

The results of this exercise reveal that 47 solutions require a nonzero trace to fully explain the polarities for the observed amplitudes, while the polarities for the remaining 19 events can be explained just as well through a purely deviatoric source. As expected, the match between...
observed and predicted amplitudes is generally better for the full moment tensor solutions (correlation coefficients range between 0.54 and 1.18) than for the constrained solutions (correlation coefficients range between 0.42 and 1.32), as the full solutions have an additional degree of freedom compared to the constrained solution. Occasionally, the change in the correlation coefficient is big enough that it could be used to favor one solution over the other but, in general, we observe that the change is not significant. The condition number for the solutions with an uncertain volumetric component ranges between 0.05 and 0.17, similar to the range observed for the full moment tensor solutions for the whole data set. As argued before, we think that this range of values reflects well-constrained moment tensor solutions, as the similarity suggests uncertain volumetric contributions are the result of uncertainties in the amplitude measurements rather than an insufficient coverage of the focal sphere. The deviatoric P-wave radiation patterns for these moment tensor solutions are displayed in Figure 9, superimposed on the Savuka mine plan.

McGarr (1992b) reported a bimodal distribution of the volumetric-shear mix for 10 events with moment magnitudes in the 1.9–3.3 range recorded in two deep mines in South Africa. The distribution peaked at values of 0.0 and 0.71 for the volumetric to shear-slip ratio ($\Delta V/\Sigma AD$), with no events having ratios between 0.0 and 0.5. Following McGarr (1992b), the volumetric closure ($\Delta V$) and the average shear slip ($\Sigma AD$) are calculated as

$$\Delta V = \frac{\text{tr}(\sigma)}{(\lambda + 2\mu)},$$

$$\Sigma AD = \frac{|\sigma_1^v| + |\sigma_2^v|}{2\mu},$$

(14)

where $\text{tr}(\sigma)$ is the trace of the moment tensor, $\sigma_1^v$ and $\sigma_2^v$ are the eigenvalues of the major and minor double couples of the deviatoric moment tensor, $\lambda$ and $\mu$ are Lamé’s elastic constants, and $A$ and $D$ represent the fault area and average slip, respectively. We have assumed that $\lambda + 2\mu = 1.63 \times 10^{11} \text{ Pa}$ and $\mu = 3.76 \times 10^{10} \text{ Pa}$ for the Witswatersrand quartzite (McGarr, 1992b). The $-\Delta V$, $\Sigma AD$, and $-\Delta V/\Sigma AD$ values for the 76 moment tensors can be found...
Figure 9. Spatial distribution of the deviatoric focal mechanisms for the Savuka mine events considered in this study (only those events with a well-constrained deviatoric part have been included). The black symbols represent those events with a significant volumetric component (more than 10% of the total scalar moment); the gray symbols represent those events with a near-deviatoric solution (less than 10% of the total scalar moment). The mine plan is the same as in Figure 1, with active areas highlighted.
Note that negative values of the source-mix ratio indicate an explosive volumetric component, null values indicate a purely deviatoric source, and positive values indicate an implosive volumetric component. A quick inspection of the ratios reveals a more or less continuous variation of volumetric closure to average shear-slip ratios ($-\Delta V/\Sigma AD$), with values ranging between $-0.4$ and $0.9$, and peaking around $0.2$ and $0.6$ (Fig. 10). The moment tensors with an uncertain volumetric contribution include all the explosive sources.

The magnitude dependence of the volumetric closure to shear-slip ratios is investigated in Figure 11. The figure includes the ratios for the 76 events reported in this study, plus the 10 measurements obtained by McGarr (1992b). The diagram shows that the distribution of volumetric to shear-slip ratios is continuous for moment magnitudes below 2.2. If we remove the poorly constrained events from our database, we observe that the ratios for events with moment magnitudes below 2.2 still display a more or less continuous range of values for the $-\Delta V/\Sigma AD$ ratio. For moment magnitudes above 2.2 the distribution seems bimodal, as reported by McGarr (1992b), but this magnitude range is clearly undersampled by our measurements.

Summarizing, the moment tensor solutions obtained in this study for 76 mine tremors in South Africa have demonstrated that the distribution of coseismic volumetric closure to shear-slip ratios is continuous in the $0.5 < M_w < 2.2$ moment magnitude range. A continuous distribution of the volumetric-shear mix had been previously reported for mine tremors at overlapping and lower magnitude ranges in shallow mines in Canada and Utah (Feignier and Young, 1992; Fletcher and McGarr, 2005), which suggests the continuous distribution of the volumetric-shear mix is independent of mining conditions. At moment magnitudes above 2.2, our measurements do not have enough overlap with those of McGarr (1992b) to either confirm or call into question their postulated bimodal distribution. The number of measurements in this magnitude range is significantly smaller than at lower magnitudes and more measurements are required to confidently assess the bimodality of the source-mix distribution.

Data and Resources

Seismic waveforms used in this study were obtained as part of the Mine Seismicity Project of AfricaArray (http://africaarray.psu.edu, last accessed July 2009). Event locations and local magnitudes were obtained from Integrated Seismic Systems International (ISSI). The moment tensor inversion codes were developed by J. Julià. Some plots were made using the Generic Mapping Tools, version 4.3.3 (http://gmt.soest.hawaii.edu [last accessed July 2009], Wessel and Smith, 1998).

Acknowledgments

AngloGold Ashanti is thanked for sharing the proprietary data for Savuka mine used in this study. We are also indebted to Integrated Seismic Systems International for their role in providing the in-mine data analyzed in this study. The manuscript has greatly benefited from detailed and insightful reviews from two anonymous referees, as well as from comments by Steve Spottiswoode and Alex Milev at the Council for Scientific and Industrial Research in South Africa. Roger Stewart is thanked for helping produce a printable version of the Savuka mine plans. Support for this work has been provided by the U.S. Department of Energy, Contract Number DE-FG52-06NA27320.

References


Electronic Supplement to

Source Mechanisms of Mine-Related Seismicity, Savuka Mine, South Africa

by J. Julià, A.A. Nyblade, R. Durrheim, L. Linzer, R. Gök, P. Dirks, and W. Walter

Focal Parameters for Mine Tremors at Savuka

The supplementary material provided here consists of four space delimited ASCII tables containing origin times, locations, moment tensor solutions, and scalar moments for the mine tremors analyzed in this study.

Tables

Table E1 [ASCII Text File; 8 KB]. Origin times, hypocentral locations, and local magnitudes for the mine tremors analyzed in this study. The origin times are given in local South African time and the locations are given in a local mine-coordinate system where x is North, y is East, and z is Up. Event locations and origin times were reported by Integrated Seismic Systems International (ISSI).

Table E2 [ASCII Text File; 12 KB]. General moment tensor solutions for the 81 events analyzed in this study. The moment tensor components are given in the local mine-coordinate system, where x is North, y is East, and z is Up. The last two columns correspond to the condition number of the singular value spectrum (cond) and the correlation coefficient between observed and predicted waveforms (c12), respectively.

Table E3 [ASCII Text File; 12 KB]. Eigenvalues (s1, s2, s3) and eigenvectors (e1, e2, e3) for the moment tensor solutions listed in Table E2. Note the eigenvalues have been sorted so that s1 < s2 < s3.

Table E4 [ASCII Text File; 8 KB]. Isotropic (mI), deviatoric (mD), and total (mT) scalar moments, and moment magnitudes (Mw) from the moment tensor solutions listed in Table E2.

Table E5 [ASCII Text File; 4 KB]. Volume closure (-DV), total shear-slip (AD), and volumetric to shear-slip ratio (-DV/AD) for the moment tensor solutions listed in Table E2.

[ Back ]